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Hadron pair photoproduction within the Veneziano model

Kosuke Odagiri

Institute of Physics, Academia Sinica, Nankang, Taipei, Taiwan 11529, The Republic of China

ABSTRACT: We first suppose that low-energy hadron pair photoproduction reactions $\gamma^{(*)}\gamma^{(*)} \to h\bar{h}$ are dominated by s-channel resonance contributions. Their normalization is then calculated by their correspondence with the Reggeon term in the Regge parametrization of the γh total cross sections. For the case of $p\bar{p}$, we make use of the measured γp total cross section, and for the case of K^+K^-, we make use of the corresponding total cross section that is estimated using Regge factorization. For hadrons that have no such data, we can only provide rough estimation based on the additive quark rule. As an effective approach that is convenient and parameter-free, we adopt the Veneziano model in the simplest form. The model is only applicable to the region of low centre-of-mass energy. When the transverse momentum is large, perturbative QCD takes over, whereas in the Regge region, it is known that the Regge pole picture fails in photoproduction. Despite the shortcomings of the model, we find that the parameter-free amplitudes offer a sound description of the data at hand.

KEYWORDS: nef.pmo.

Contents

1. Introduction

1.1 Experimental status

Belle has accumulated large statistics in hadron-pair photoproduction event samples, in particular, of $\gamma^{(*)}\gamma^{(*)} \to p\bar{p}$ [1] and K^+K^- [2]. In both cases, the measured cross section and the angular distribution are largely in agreement with the previous measurements [3, 4, 5, 6]. As an example, we show the results[1] [7] for the $p\bar{p}$ measured total cross section in fig. 1 and the angular distribution in fig. 2. We should note that the difference between the Belle measurement and the other experiments has been resolved and the latest results [1] are largely in agreement with the data from VENUS [3] and CLEO [4].

The Belle measurements are restricted to the central region defined by $|\cos\theta^*| < 0.6$, where θ^* is the polar angle in the centre-of-mass frame of the produced hadron pair. The total cross section in this region as a function of the effective centre-of-mass energy $W_{\gamma\gamma}$ shows prominent resonance structure for the case of K^+K^-, and a near-continuum structure for the case of $p\bar{p}$ although some remnants of individual resonances can still be

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[1]We thank C.C. Kuo for their kind permission to reproduce their figures in this paper. The discussions we present in this paper are, however, based on a more recent analysis of ref. [1].
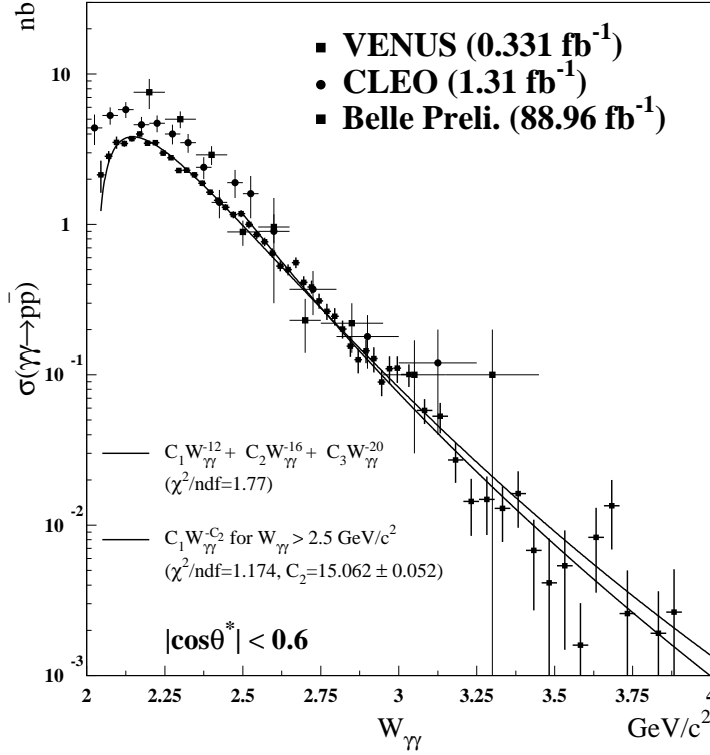
Figure 1: $\gamma\gamma \to p\bar{p}$ cross section versus centre-of-mass energy at VENUS, CLEO and Belle in the central region, defined by $|\cos\theta^*| < 0.6$. The vertical error-bars on the Belle data are due to the statistical error in the event and the Monte Carlo samples only.

seen. This is in accord with the expectation from the fact that the proton is heavier than the kaon and that the mass difference between resonances is small when $W_{\gamma\gamma}$ is large.

The angular distributions show a more marked trend. At sufficiently high $W_{\gamma\gamma}$, the events are concentrated in the forward region as expected on general grounds from t-channel exchange be it of perturbative or nonperturbative nature, but just above the threshold, the distributions are peaked in the transverse direction, i.e., $\cos\theta^* \sim 0$. This latter behaviour would be very difficult to explain in a perturbative analysis but is easily accommodated in the resonance picture, as follows.

Let us consider the simpler case of the reaction $\gamma\gamma \to K^+K^-$, and suppose that it proceeds by an s-channel spin-2 resonance, for example $f_2(1270)$, $a_2(1320)$ or $f_2'(1525)$. As $K^\pm$ are spin-0, there are altogether only two independent helicity combinations, due to the initial-state photon helicities. The net s-channel helicity is either ± 2 or 0 along the direction of the photons. The angular distributions are then proportional to the square of:
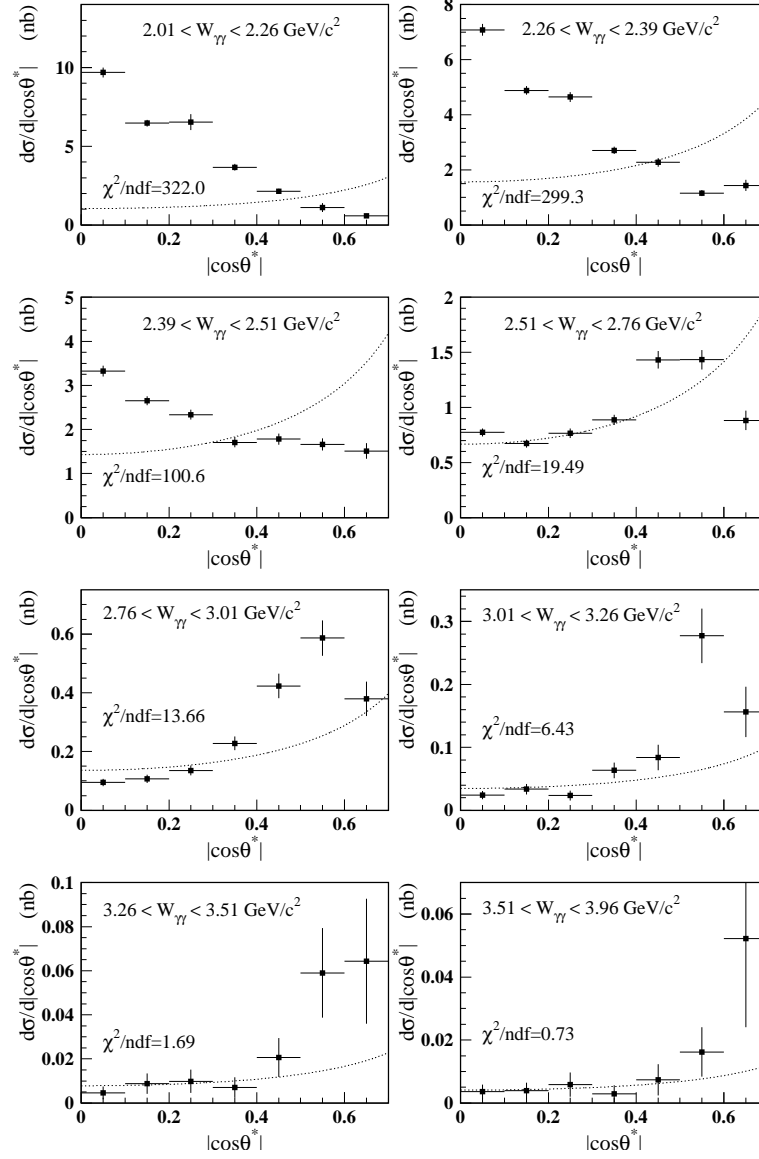
$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta^*, \tag{1.1}$$

Figure 2: Angular distribution of $\gamma\gamma \to p\bar{p}$ events at Belle. The vertical error-bars represent the statistical uncertainty only.

$$d_{0,0}^2 = \frac{3}{2}\cos^2\theta^* - \frac{1}{2}. \tag{1.2}$$

For real photons, the two amplitudes do not interfere because of different photon helicities. We thus see that the distribution has a local maximum at $\cos\theta^* = 0$ in both cases. For the latter amplitude, the distribution also has maxima at $\cos\theta^* = \pm1$. The K^+K^- cross section just above the threshold indeed shows a marked peak corresponding to $f_2'(1525)$ and so it is inevitable that the distribution is peaked at $\cos\theta^* = 0$.

This above argument provides good reason to suppose that the process is predominantly determined by resonance dynamics in the energy range probed by Belle. However,

the $p\bar{p}$ cross section seems to have a $\sim W_{\gamma\gamma}^{-10}$ behaviour expected from the quark-counting rule and so, at first sight, seems to have a perturbative description too.

The quark-counting rule states that at high energy, the large-angle $2 \to 2$ scattering cross section is described by:

$$\frac{d\sigma}{dt}\bigg|_{\cos\theta^*=\mathrm{const.}} = \frac{f(\cos\theta^*)}{s^{K-2}}, \tag{1.3}$$

where K is the number of 'elementary fields', i.e., in our case, quarks and photons, in the external legs. For meson pair production, we have $K = 6$, and for baryon pair production, we have $K = 8$. s in this case is $W_{\gamma\gamma}^2$. Hence the cross section, integrated over $|\cos\theta^*| < 0.6$, should scale as $W_{\gamma\gamma}^{-10}$ for baryon pair photoproduction and $W_{\gamma\gamma}^{-6}$ for meson pair photoproduction.

There are two problems with this application to the case of $p\bar{p}$. First, contrary to eqn. (1.3), the angular distribution depends on $W_{\gamma\gamma}$. In fact, it is found [3, 8, 9] that in the more central region $|\cos\theta| < 0.3$, the fall-off is more rapid whereas one would expect the quark-counting rule to work better in this region. Second, fitting the data above 2.5 GeV with the power kept as a free parameter yields $W_{\gamma\gamma}^{-14.87}$ [1] rather than $W_{\gamma\gamma}^{-10}$. It seems that at 2.5 GeV, at least, the quark-counting rule is not strictly applicable.

1.2 An approach based on the Veneziano model

In the light of the above, it seems natural to consider an approach based on resonance dynamics borrowing some ideas from Regge theory [10]. For the case of $p\bar{p}$, if the f/a mesons are responsible, their contribution can be normalized through the optical theorem by finding the contribution of the f/a trajectories to the γp total cross section which is given by [11, 12]:

$$\sigma_{\mathrm{tot}}(\gamma p) = 0.0677s^{0.0808} + 0.129s^{-0.4525}. \tag{1.4}$$

The cross section is measured in mb and s is measured in GeV^2. The former term is the soft pomeron contribution and the latter term is the contribution of f_2 and a_2 trajectories, of which we know that f_2 is the dominant [11] so that we expect the γn total cross section to be almost identical.

As an economical way of writing the many resonance contributions, we propose to adopt the Veneziano model [13].

One advantage in adopting the Veneziano model is that the amplitude is dual by construction, and so, in principle, the transition from the s-channel resonance picture to the t-channel pole picture should be consistent and suffer from no double-counting as would be the case had we simply summed the s-channel and t-channel contributions together.

Having said that, there are two pitfalls that one should bear in mind throughout.

First, the Veneziano model is constructed for spin-less meson scattering processes. The model would perform badly where the helicities of the particles matter. For example, for baryon exchange processes, there is incorrect behaviour in the Regge limit as compared with the Regge pole expectations.

Second, photoproduction processes, such as $\gamma p \to \pi^0 p$, are known to violate the simple Regge pole picture. Instead of the usual $s^{\alpha(t)}$ behaviour of the amplitude in the Regge

limit, i.e., large s and small fixed t, it has been found that $d\sigma/dt$ goes as s^{-2} [11] when meson exchange is expected and s^{-3} [14] when baryon exchange is expected. In both cases, no sign of 'shrinkage' is found, i.e., the amplitude, as a function of t, goes not as $s^{\alpha(t)}$ but as $C^{\alpha(t)}$ where C is constant. It is worth noting here that the other baryon exchange processes are also known to share these same characteristics [14].

We hence see that our picture should work only at low $W_{\gamma\gamma}$ where the perturbative picture is still not applicable and, at the same time, we are safely away from the Regge region where the Regge pole picture presumably breaks down. This last point, however, merits further experimental investigation.

In addition, even when $W_{\gamma\gamma}$ is small, there would be problems when the helicity structure is important. One other point to bear in mind is that the resonances do not lie exactly on linear trajectories and so there would be some error coming from inaccurate parametrization. We should emphasize here that what we are dealing with is just a simple 'toy model' of resonance-and-pole-dominated physics, and in any case the economy of the model is sufficiently attractive to merit the analysis of its instigation.

We perform several simple calculations to show that despite all these restrictions, the result shows striking resemblance to real data, though the quantitative description leaves room for improvement.

The paper is organized as follows. We write down the amplitude in sec. 2. The result of calculations is shown in sec. 3, together with the discussion of our findings. The conclusions are stated at the end.

2. The Veneziano amplitude

The Veneziano amplitude [13] is an amplitude with the pole structure inspired by Regge theory and conforms to the idea of duality [10]. For meson trajectories, it reproduces the Regge behaviour at fixed t in the limit of large s. For baryon trajectories this does not work and, if we desire to recover Regge behaviour we must, for example, insert an extra $1/\sqrt{s}$ factor in the amplitude.

However, it is known that processes involving baryon exchange are not described well by the Regge pole picture [14]. Furthermore, photoproduction processes are not described well by the Regge pole picture even when mesons are exchanged, as in the process $\gamma p \to \pi^0 p$ [11].

2.1 The formalism

Let us write down our amplitudes as follows. Meson pair photoproduction is given by:

$$A(\gamma\gamma \to M\overline{M}) = \frac{\overline{\beta}}{\pi} \frac{\Gamma(1-\alpha_s(s))\Gamma(1-\alpha_t(t))}{\Gamma(1-\alpha_s(s)-\alpha_t(t))} + (t \leftrightarrow u). \tag{2.1}$$

Baryon pair photoproduction is given by:

$$A(\gamma\gamma \to B\overline{B}) = \frac{\overline{\beta}}{\pi} \frac{\Gamma(1-\alpha_s(s))\Gamma(\frac{1}{2}-\alpha_t(t))}{\Gamma(\frac{1}{2}-\alpha_s(s)-\alpha_t(t))} + (t \leftrightarrow u). \tag{2.2}$$

Γ is the Euler Gamma function. $\alpha_s(s) = \alpha_s(0) + \alpha'_s s$ is the s channel trajectory (and certainly should not be confused with the strong coupling α_S to which it has no direct relation). $\alpha_t(t) = \alpha_t(0) + \alpha'_t t$ is the t channel trajectory. Both trajectories need to be linear. $\overline{\beta}$ is the overall coupling, which we may assume to be constant for simplicity.

In order to obtain the leading contribution, one should set $\alpha_t(t)$ to be the leading trajectory, i.e., the trajectory with the greatest intercept $\alpha_t(0)$, consistent with the t-channel quantum numbers. For the s-channel, one should include all trajectories that have significant contribution. We have assumed in the above that the lightest member of the s-channel trajectory is spin-1 and the lightest member of the t-channel trajectory is spin-1 or spin-1/2. However, the leading s-channel contribution is spin-2 by symmetry after summing the t-channel and the u-channel contributions together, so that there is no contribution from, for example, ρ and ω.

2.2 Remarks on the amplitude

We note that the $s \leftrightarrow t$ crossed amplitude corresponding to eqn. (2.2) does not have correct angular behaviour expected from fermionic resonances [14]. This is the reason for the incorrect Regge behaviour in the uncrossed amplitude.

The Regge limit, $s \to \infty, t$ or $u =$const., for the above amplitudes is obtained using the Stirling formula in the form:

$$\Gamma(x+a)/\Gamma(x+b) = x^{a-b}\left(1 + \mathcal{O}(x^{-1})\right). \tag{2.3}$$

For the two amplitudes, we obtain respectively:

$$A(\gamma\gamma \to M\overline{M}) \approx \frac{\overline{\beta}}{\pi}\Gamma\left(1 - \alpha_t(t)\right)(-\alpha_s(s))^{\alpha_t(t)} + (t \leftrightarrow u), \tag{2.4}$$

$$A(\gamma\gamma \to B\overline{B}) \approx \frac{\overline{\beta}}{\pi}\Gamma\left(\frac{1}{2} - \alpha_t(t)\right)(-\alpha_s(s))^{1/2+\alpha_t(t)} + (t \leftrightarrow u). \tag{2.5}$$

Only the former has the behaviour expected from Regge theory. The latter has an extra $1/2$ in the power.

The mathematical properties of the Veneziano amplitude are well documented elsewhere [10, 11, 13], but it is worth restating the manifestation of duality in it. The Veneziano amplitude is dual in the sense that we may expand it either in terms of the s-channel resonances or the t-channel poles, and when expanded in terms of the s-channel resonances, no t-channel poles appear, and vice versa. For the Euler beta function $B(x, y)$, following the notation of ref. [11], we have:

$$B(x,y) \equiv \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \equiv \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{x+n}\frac{\Gamma(y)}{\Gamma(y-n)} \equiv \sum_{n'=0}^{\infty}\frac{(-1)^{n'}}{n'!}\frac{1}{y+n'}\frac{\Gamma(x)}{\Gamma(x-n')}. \tag{2.6}$$

We see that when expanded in terms of poles in x, no poles in y appear, and vice versa. The amplitudes (2.1), (2.2) are related trivially to $B(x, y)$ and so no double poles appear here also. We express this diagrammatically, for the baryonic case, in fig. 3.

In fig. 3, in addition to the terms included in eqns. (2.1) and (2.2), a third class of terms is shown, which are $t - u$ dual. We do not consider this class of terms, because we wish to
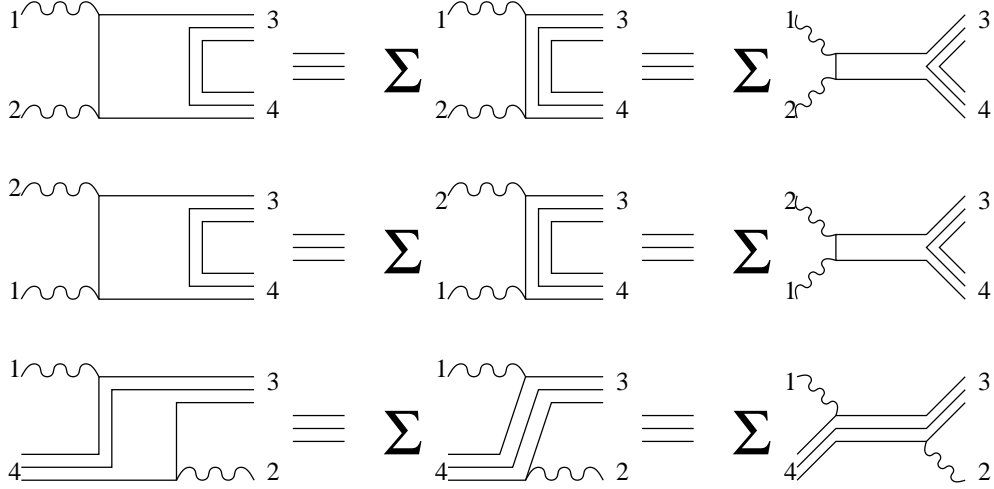
Figure 3: The quark-line diagrams for the three dual amplitudes in $\gamma\gamma \to p\bar{p}$. The diagrams shown are for the $s - t$ amplitude (top), the $s - u$ amplitude (middle) and the $t - u$ amplitude (bottom).

consider only terms that have a s-channel description and because we know that the Regge t-channel pole picture fails in photoproduction. For the same reason, we should emphasize that although the amplitudes as written above are expected to entail a smooth transition from a low-energy s-channel dominated picture to the Regge t-channel pole picture, one should regard this only as an indication of the enhancement in the forward region and not as a prediction of how the forward amplitude should rise.

We also note that the pole expansion in eqn. (2.6) requires the existence of daughter trajectories that contain hadrons with approximately the same masses as the hadrons on the parent trajectory and lower J.

2.3 The trajectories

The trajectories that we consider are as follows. For the s-channel a/f trajectories, we adopt:

$$\alpha_{a/f}(s) = 0.5 + 0.4i + 0.9s. \tag{2.7}$$

For the real part of the Reggeon slope and intercept, we used the values suggested in ref. [11] that applies to the case when $\rho/\omega/a/f$ are all on a universal trajectory. Without this constraint, ref. [11] suggests $0.70 + 0.80s$ for the a/f trajectories. The imaginary part has been chosen so as to yield a width of approximately 200 MeV for resonances at about 2 GeV. s is measured in GeV^2 as mentioned before. As resonances occur for positive integer values of $\alpha(s)$, the resonances are at:

$$M_{a/f} = \sqrt{(n - 0.5)/0.9} \text{ GeV}, \quad (n = 1, 2, 3, \ldots). \tag{2.8}$$

We note that although the resonances from the leading trajectory are only manifest at even integer values of n, there are resonances for every value of n, on the daughter trajectories. We tabulate the first few resonance masses in tab. 1

n	1	2	3	4	5
M / GeV	0.745	1.291	1.667	1.972	2.236
PDG		$f_2(1270)$		$f_4(2050)$	

Table 1: The first few resonance masses on the trajectory $0.5 + 0.9s$, and the known leading resonances from ref. [15]. The masses of $f_2(1270)$ and $f_4(2050)$ are 1275.4 ± 1.2 MeV and 2025 ± 8 MeV, respectively. The assignment to the daughter trajectories can be found in ref. [11].
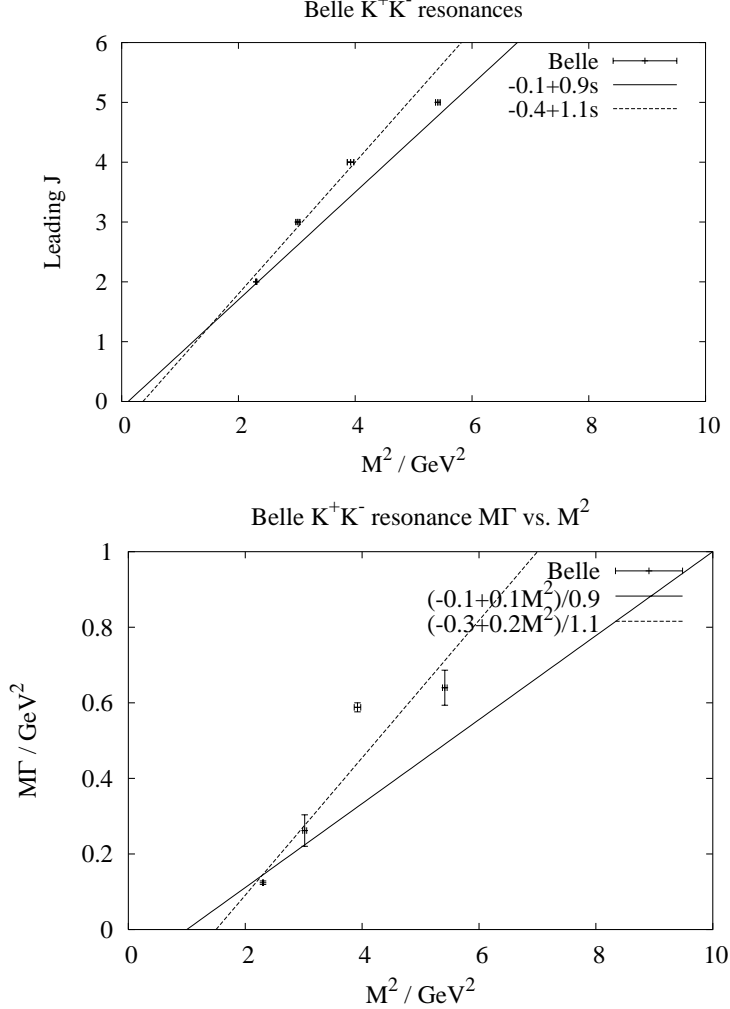

Figure 4: Resonances in $\gamma\gamma \to K^+ K^-$ found in ref. [2]. The first plot (above) shows a 'guess' for the leading trajectory and its parametrizations, if these resonances belong to a family. The second plot (below) shows the resonance widths against our parametrizations. The solid line in each case is the parametrization adopted for simulation.

In addition, for processes that involve strange quarks, we adopt:

$$\alpha_{f'}(s) = -0.1 - 0.1i + (0.9 + 0.1i)s. \tag{2.9}$$

The parametrization is demonstrated in fig. 4, together with another possibility corre-

sponding to $\alpha_{f'}(s) = -0.4 - 0.3i + (1.1 + 0.2i)s$. The values for J shown in fig. 4 are merely guesses for the leading members in the trajectory, and not a result of experimental analysis. The odd-valued J would correspond to the ϕ mesons on the parent trajectory.

The real part was chosen to go through $f_2'(1525)$ with the constraint that the slope is 0.9. This roughly follows the resonances found in ref. [2], but it is not the best choice, as can be seen in fig. 4.

The widths also approximate the values found in ref. [2], but this is again not the best choice. Contrary to eqn. (2.7), the imaginary part in this case has a slope. This is necessary since $f_2'(1525)$ has a relatively small width of about 80 MeV whereas the heavier resonances are expected to have greater widths.

We should bear in mind that the analysis of ref. [2] does not fully take into account the possibility that there are further contributions which have peak positions near these resonances, such as there is in our present analysis from a and f mesons. Hence the masses and widths measured therein are not conclusive. This is the reason why we chose to put less weight on tuning with the resonances of ref. [2] and used a parametrization which, in the author's opinion, is more conservative than the other choice shown in fig. 4.

n	1	2	3	4	5
M / GeV	1.106	1.528	1.856	2.134	2.380
Belle		1.518	1.737	1.980	2.327
PDG	$\phi(1020)$	$f_2'(1525)$	$\phi_3(1850)$		

Table 2: The first few resonance masses on the trajectory $-0.1 + 0.9s$, compared with the result of the Belle analysis and with some of the ϕ/f' resonances from ref. [15].

For the sake of comparison, the first few resonance masses according to the parametrization of eqn. (2.9) are tabulated in tab. 2 and are compared against the Belle analysis and some of the ϕ/f' resonances from PDG [15]. There is also a $\phi(1680)$ in PDG which is difficult to assign. On the other hand, the masses of $f_2'(1525)$ and $\phi_3(1850)$ are in agreement with our parametrization.

For the t-channel trajectories, we have for the leading Δ/N trajectory:

$$\alpha_{\Delta/N}(t) = 0.0 + 0.9t, \tag{2.10}$$

both for the neutral and charged baryons. This is taken from ref. [14].

For $K^\pm$, the leading trajectory is that containing $K^*(892)$ rather than $K^\pm$ whose mass is 493.677 ± 0.016 MeV, and we write this as:

$$\alpha_{K^*}(t) = 0.2 + 0.9t. \tag{2.11}$$

The numbers were fixed here by assuming that the slope is 0.9, the same as in eqn. (2.7). Having said that, this is in good agreement with the known K mesons, as shown in fig. 5.

For Λ and Σ baryons, we have the leading Σ trajectory given by:

$$\alpha_\Sigma(t) = -0.27 + 0.9t. \tag{2.12}$$
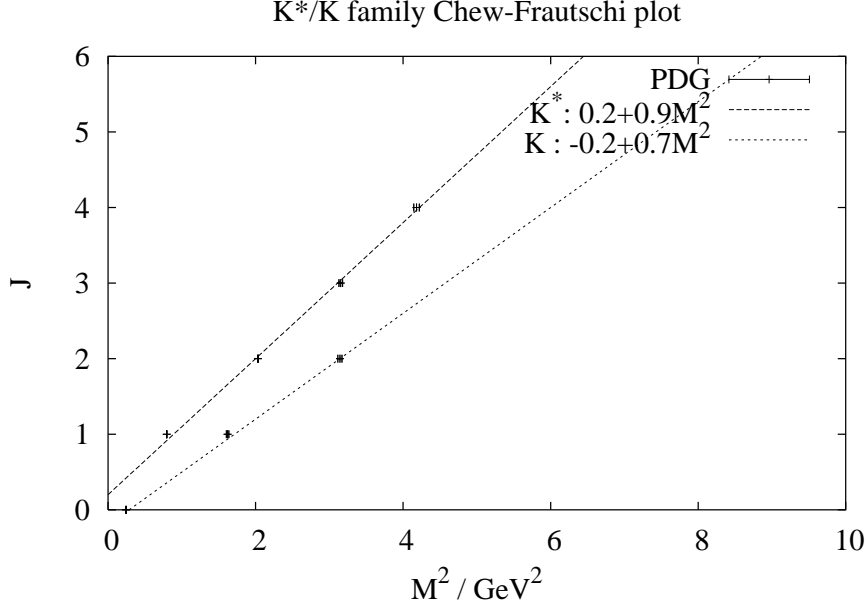
This is again taken from ref. [14].

Figure 5: Leading K and K^* trajectories.

2.4 The normalization

Let us now find the normalization factor $\overline{\beta}$ by considering the Reggeon mediated part of the photon-hadron total cross sections. The total cross sections are related to the imaginary parts of the elastic scattering amplitudes by the optical theorem:

$$\sigma_{\text{tot}}(\gamma h) = \frac{1}{s}\text{Im}\left[A(\gamma h \to \gamma h)\right]\Big|_{t=0}. \tag{2.13}$$

We have not explicitly introduced the spin indices but the averaging over the amplitudes is implicit. In this case, the amplitudes should be understood to be the average of the amplitudes in which the γ and h helicities are separately conserved. The helicity-flipping amplitudes can not be estimated using our present method.

After crossing the amplitudes (2.1), (2.2) to obtain the relevant $\gamma h \to \gamma h$ amplitudes, we take the Regge limit of fixed t, in this case $t = 0$, and large s and make use of eqn. (2.3). Both eqns. (2.1) and (2.2) have the correct Regge behaviour in this limit, and we obtain:

$$\sigma_{\text{tot}}(\gamma h) \approx \frac{\overline{\beta}}{\pi s}\Gamma\left[1 - \alpha_s(0)\right]\left(\alpha'_t s\right)^{\alpha_s(0)}\sin\left[\pi\alpha_s(0)\right]. \tag{2.14}$$

We note that there is in reality an additional contribution from the pomeron which dominates in the large $\sqrt{s}$ limit. Here we limit ourselves to the discussion of the a/f meson contributions only. From the real part of eqn. (2.7), we have the intercept $\alpha_s(0) = 0.5$. For $\alpha'_t = 0.9$ and converting from GeV^2 to mb, we obtain:

$$\sigma_{\text{tot},\,a/f}(\gamma h) \approx 0.208\overline{\beta}s^{-0.5}\text{ mb}. \tag{2.15}$$

For $\gamma\gamma \to p\bar{p}$, $\overline{\beta}$ can then be determined by comparing against the second term of eqn. (1.4), namely $0.129s^{-0.4525}$ mb. However, the powers are slightly different due to the different

values of the a/f intercept adopted. We choose to equate the two at 10 GeV and obtain:

$$\overline{\beta}(\gamma\gamma \to p\bar{p}) = 0.770 \ . \tag{2.16}$$

We note that the $n\bar{n}$ process should be almost identical to the $p\bar{p}$ case. The pn and $\bar{p}n$ total cross sections are similar to the pp and $\bar{p}p$ total cross sections, indicating that the isospin-1 contribution from the a trajectory is small.

The differential cross section is given generally by:

$$\frac{d\sigma}{d\cos\theta^*} = \frac{\sqrt{1 - 4m_h^2/s}}{32\pi s}\frac{1 + 2J_h}{2}\frac{1}{1 + \delta}\,|A|^2 \ . \tag{2.17}$$

m_h is the hadron mass and J_h is the spin. δ is 1 for identical particle final states, for example $\gamma\gamma \to \pi^0\pi^0$, and 0 otherwise.

We have again assumed that only the helicity conserving amplitudes contribute in the sense that, for example, the initial-state photons are in the left–right and the right–left combinations, as the other amplitudes can not be estimated.

t and u are calculated from s and $\cos\theta^*$ as usual by:

$$t = m_p^2 - \frac{s}{2}\left(1 - \cos\theta^*\sqrt{1 - 4m_p^2/s}\right), \quad u = m_p^2 - \frac{s}{2}\left(1 + \cos\theta^*\sqrt{1 - 4m_p^2/s}\right). \tag{2.18}$$

Now let us turn our attention to the case of $\gamma\gamma \to K^+K^-$. There is no data for the $\gamma K^\pm$ total cross section, so we must estimate it using Regge factorization using the $pK^\pm$ scattering total cross section, which goes as [11, 12]:

$$\sigma_{\text{tot}}(K^+p) = 11.93s^{0.0808} + 7.58s^{-0.4525} \text{ mb}, \tag{2.19}$$

$$\sigma_{\text{tot}}(K^-p) = 11.93s^{0.0808} + 25.33s^{-0.4525} \text{ mb}. \tag{2.20}$$

In both of the above, the first term corresponds to the pomeron contribution. The second term is due to the $\rho/\omega/a/f$ trajectories. The difference in the two cases is due to the $C = -1$ trajectories, i.e., the ρ and the ω, contributing in the opposite way. Thus the a/f contribution that we are interested in can be obtained as the mean of the two, i.e., 17.255. The same in the case of pp and $p\bar{p}$ cross sections gives 77.235.

We then calculate the $\gamma K^\pm$ total cross section by assuming that the couplings to the trajectories factorize. We use our knowledge of the $\gamma K^\pm$, γp, pp and $p\bar{p}$ cross sections to write:

$$\sigma_{\text{tot}}(\gamma K^\pm) \approx \frac{11.93 \times 0.0677}{21.7}s^{0.0808} + \frac{17.255 \times 0.129}{77.235}s^{-0.4525} \text{ mb}$$
$$= 0.0372s^{0.0808} + 0.0288s^{-0.4525} \text{ mb}. \tag{2.21}$$

This time, we obtain from eqn. (2.15):

$$\overline{\beta}(\gamma\gamma \to K^+K^-) \approx 0.172. \tag{2.22}$$

There is an additional complication in the case of kaons because of the presence of the strange quark. We expect, as is evident in the data [2], a significant contribution from

the f' trajectory. The coupling of the f' trajectory can only be estimated. For the sake of argument, let us say here that the coupling is proportional to the square of the electric charge of the strange quark, i.e., $e^2/9$. On the other hand, $f/a \sim (u\bar{u} + d\bar{d})/\sqrt{2}$ so the average charge squared is 2.5 times that of the strange quark. Hence we estimate $\overline{\beta}$ to be 2.5 times less than that of the f/a trajectory. We shall see later that this leads to a not unreasonable description of the cross section.

For the sake of completeness, let us calculate the case of $\pi^\pm$ production. In this case, taking the $\pi^\pm p$ scattering total cross sections as input, factorization yields:

$$\sigma_{\text{tot}}(\gamma\pi^\pm) \approx \frac{13.63 \times 0.0677}{21.7}s^{0.0808} + \frac{31.79 \times 0.129}{77.235}s^{-0.4525} \text{ mb}$$
$$= 0.0425s^{0.0808} + 0.0531s^{-0.4525} \text{ mb}. \tag{2.23}$$

Hence, again from eqn. (2.15) and assuming that the slope of the pion trajectory is also 0.9, we obtain:

$$\overline{\beta}(\gamma\gamma \to \pi^+\pi^-) = 0.318. \tag{2.24}$$

2.5 The additive quark rule

When considering hadrons other than $p, n, \pi^\pm$ and $K^\pm$, the above prescription fails due to the lack of data. In this case, the best that we can do is to estimate the coefficients by the additive quark rule.

As noted in ref. [12], the additive quark rule is a good rule-of-the-thumb for the pomeron. We can say that the pomeron couples with equal strengths to the u-quark and the d-quark, and with 70% of the strength to the s-quark. This gives good estimation for $p\pi^\pm$ and $pK^\pm$ scattering cross sections. However, this does not work for the a/f couplings.

For example, we see from the above that $\overline{\beta}$ is about 0.318 for $\gamma\gamma \to \pi^+\pi^-$ and is about 0.172 for $\gamma\gamma \to K^+K^-$. Since there are two u/d quarks in the pion and only one in the kaon, it seems that $0.318/0.172 \sim 2$ is in accordance with expectation. The problem is that from the same line of thought, we would expect $\overline{\beta}$ for $\gamma\gamma \to p\bar{p}$ to be about 1.5 times the pionic case, and this is incorrect as we have 0.770 in this case rather than ~ 0.48.

However, for the sake of rough estimation, we can proceed by applying the additive quark rule separately to the mesons and the baryons. We then have, for baryons:

$$\overline{\beta}(\gamma\gamma \to \Lambda\overline{\Lambda}) \approx \overline{\beta}(\gamma\gamma \to \Sigma\overline{\Sigma}) \approx \frac{2}{3}\overline{\beta}(\gamma\gamma \to p\bar{p}). \tag{2.25}$$

In our approximation, for the case of Σ, only the f trajectory, and not the a trajectory, contributes.

We should additionally consider the coupling of the f' trajectory to the s-quark content of Λ and Σ. We can again make a rough estimate of this by considering the squared electric charge of the s-quark, and say that $\overline{\beta}$ in this case is about one fifth of the $\overline{\beta}$ for the f trajectory. We assume that the trajectory of eqn. (2.9) holds also at the baryonic mass scale.

One possibly severe drawback of this approach is that we have not taken into account the difference in isospin between Λ, with $I = 0$, the proton, with $I = 1/2$, and Σ^0, with

$I = 1$. In particular, Λ is exchange anti-symmetric between u and d so we expect that the exchange symmetric, i.e., isospin-0, f trajectory should not couple to the Λ. We know that the isospin-1 a trajectory has much smaller coupling [11] to p and n than the f trajectory, so that we would expect the $\gamma\gamma \to \Lambda\overline{\Lambda}$ cross section to be much smaller than is implied by eqn. (2.25).

We must turn to the data in order to resolve this point. The data from L3 [16] seem to suggest that the Λ pair production cross section, at the level of the total cross section, is comparable to the Σ^0 pair production cross section. However, we should wait for improved statistics in order to confirm this point, and we should note furthermore that there is possibly discrepancy in the data on Λ pair photoproduction between L3 [16] and CLEO [17] which needs to be understood.

Let us here proceed by taking the additive quark rule literally as the rule-of-the-thumb governing the sum of the couplings of the f and the a trajectories. As we shall see in the following section, this in fact leads to a greater total cross section for Λ than Σ, indicating the need to take into account the suppression due to isospin.

There is no difference between Σ^-, Σ^0 and Σ^+. Since their masses and the trajectories are also similar, we have:

$$\sigma(\gamma\gamma \to \Sigma^+\overline{\Sigma^+}) \approx \sigma(\gamma\gamma \to \Sigma^0\overline{\Sigma^0}) \approx \sigma(\gamma\gamma \to \Sigma^-\overline{\Sigma^-}). \tag{2.26}$$

As mentioned above, the same relation holds between the proton and the neutron cross sections. These relations are independent of the additive quark rule, and are also independent of the formalism that we have adopted. Any violation of these equalities would indicate that there is significant contribution from the isospin-1 resonances, and this would seem implausible from the similarity of the $pn, \bar{p}n$ total cross sections with the $pp, \bar{p}p$ total cross sections.

We note that another possible source of violation of this flavour symmetry would be the third term of fig. 3, which is also expected to be small except in the forward region.

Our predictions are thus in contrast with those of ref. [18], in which the exchange symmetry is between the d- and the s-quarks rather than the u- and the d-quarks.

3. Result and discussions

Following the above discussions of the formalism and parametrization, let us now proceed to examining the results of our calculation for the cases of $\gamma\gamma \to p\bar{p}$ and $\gamma\gamma \to K^+K^-$, after which we turn to the case of the other baryons.

3.1 $\gamma\gamma \to p\bar{p}$

We first show the result of calculation for the $\gamma\gamma \to p\bar{p}$ cross section in figs. 6 and 7. The Belle result shown in fig. 6 corresponds to the old data points of ref. [7]. The more recent data points from ref. [1] exhibit slightly higher cross sections, and so are more in agreement with the calculation.
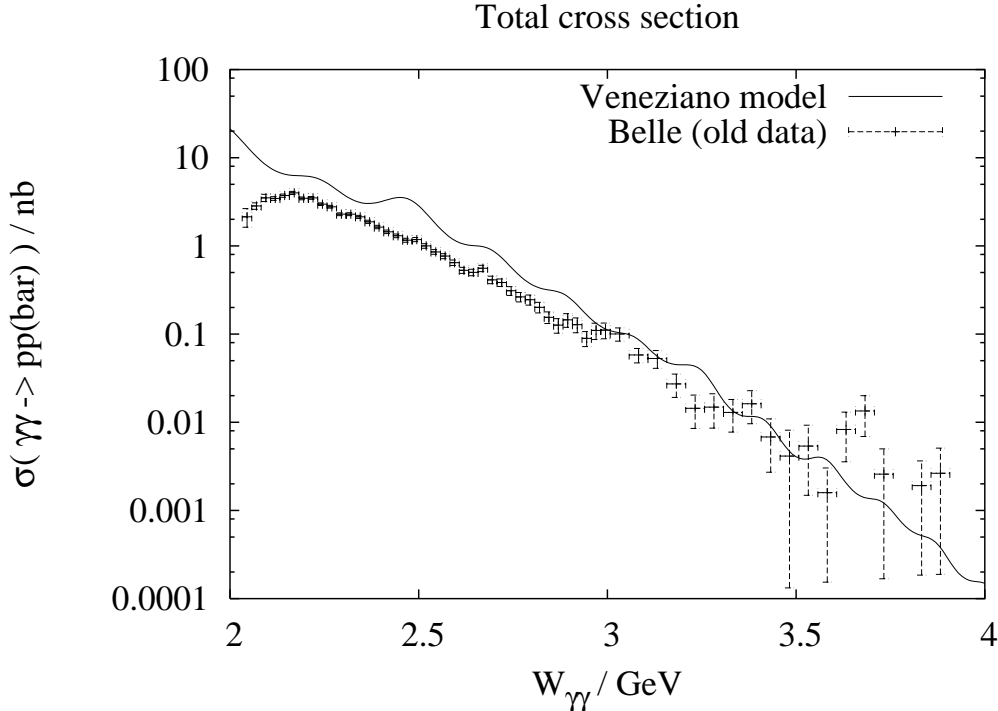
Total cross section

Figure 6: The $\gamma\gamma \to p\bar{p}$ total cross section in the region $|\cos\theta^*| < 0.6$, calculated using the Veneziano model. The Belle result is shown for comparison. The vertical error-bars represents the statistical uncertainty.

The total cross section, shown in fig. 6, shows a good $W_{\gamma\gamma}$ dependence up to about 3 GeV. This is as expected as perturbative contributions are expected to become increasingly dominant at higher energies and 3 GeV seems to be a reasonable turnover point.

We would expect that the turnover point is determined not only by $W_{\gamma\gamma}$ but also by $\cos\theta^*$ because the relevant hard scale for QCD should be approximately the transverse momentum p_T. In the more central region, we would expect that the perturbative contributions set in earlier, and the agreement of our result with the measurement would be poorer.

The resonances are more pronounced in this analysis than the real data. However, the positions of the resonances seem to match some of the fluctuations in the real data. There are extra contributions in the real data, for instance, from a peak corresponding to $\eta_c(2980)$ which is, of course, absent in the present analysis.

The threshold behaviour near 2 GeV seems wrong in the present calculation. We note that this is not definitive as the experimental statistics in this region is small, but we have a reasonable understanding as to why this is the case, which is as follows.

We know that the interactions of both the gluon and the photon with quarks are chirality conserving. Hence in the limit where the masses are small compared to the centre-of-mass energy, the interactions would predominantly conserve helicity. If this holds to some degree even in the threshold region, then the final state proton helicity would
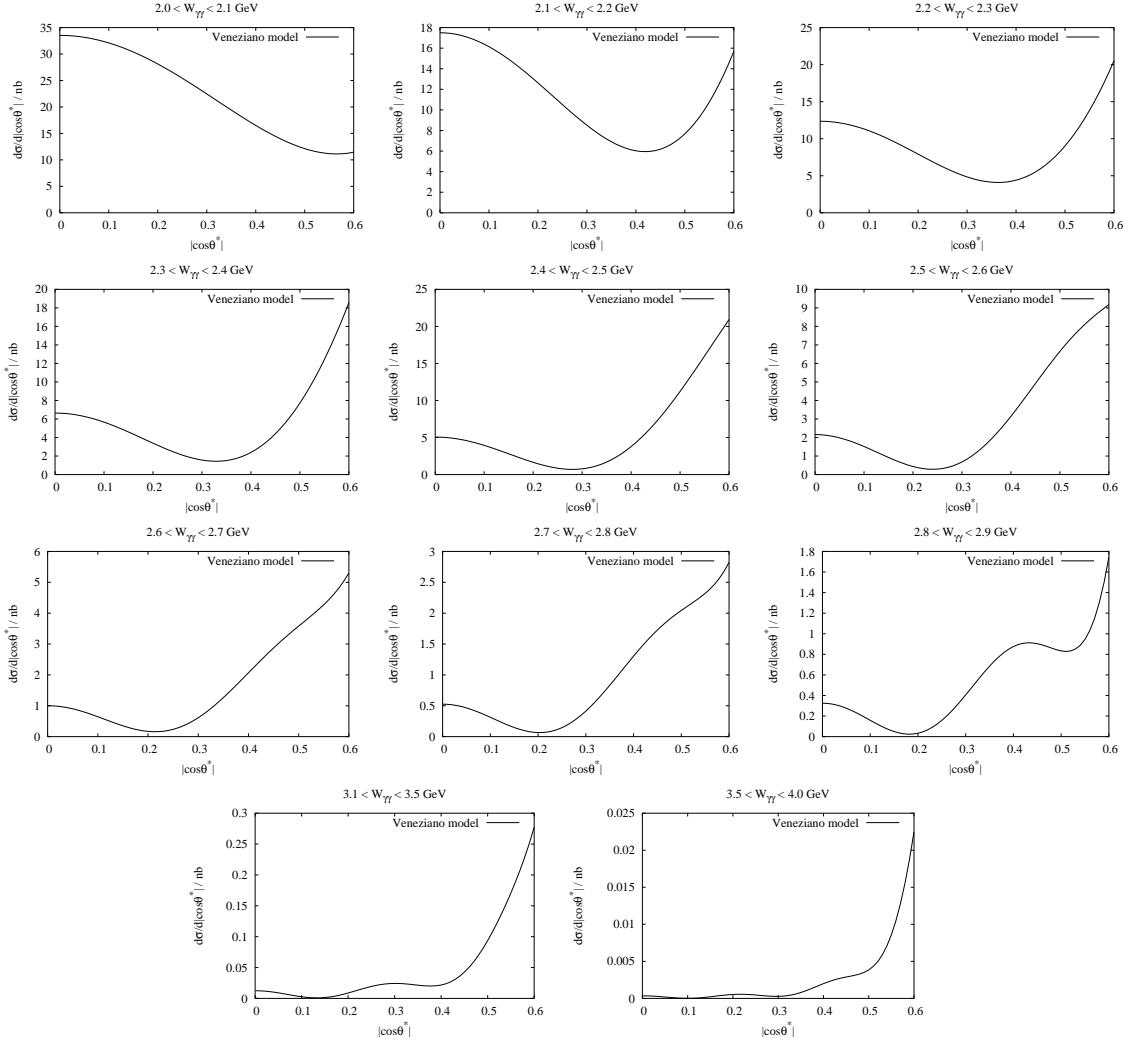
Figure 7: The angular distribution for $\gamma\gamma \to p\bar{p}$.

be opposite to the antiproton helicity here also. The total s-channel helicity along the direction of the proton would thus be ± 1. This can not be mediated by a spin-0 particle. The problem is that the amplitude as written in eqn. (2.2) contains contributions from spin-0 daughter resonances.

Just above the threshold, spin-0 terms in eqn. (2.2), i.e., the s-wave terms, are enhanced compared to the higher-spin terms by virtue of having no suppression by integer powers of $(1 - 4m_p^2/s)$ where m_p is the proton mass. This thus explains why the threshold region is described particularly badly in this approach. This also provides one possibility as to why the calculated cross section is higher than the real data below $W_{\gamma\gamma} \approx 3$ GeV.

This hypothesis can be investigated simply by analyzing the angular distribution in the threshold region[2].

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[2]We should note that spin-0 terms are not completely excluded as, for instance, the $\eta_c(2980)$ resonance is spin-0. We thank C.C. Kuo for reminding us of this point.

Turning now to the angular distributions shown in fig. 7, we see that a good qualitative description of the general behaviour is obtained. Unfortunately the positions of the dips are not in exact agreement with data.

As a general trend, the t-channel behaviour sets in earlier than in the data. This is presumably due to the combination of the two deficiencies of the model mentioned earlier, namely that the baryonic Veneziano amplitude of eqn. (2.2) has wrong forward behaviour in the Regge region, and that baryon exchange processes or photoproduction processes are in any case not described well by a simple Regge pole picture.

Nevertheless, the numbers seem more than satisfactory for a simple model with known deficiencies and no adjustable parameters.

The secondary dip seen above 2.8 GeV in the model is an interesting feature that merits further study with increased experimental statistics. Physically, this is due to the presence of resonances with higher J.

The dips seem more exaggerated in the Veneziano model compared with the real data. One explanation is the greater smearing between the peaks found in the real data in the plot of the total cross section, which in turn mixes the contributions from different resonances.

Another possible explanation is that there is further contribution from helicity-flipped contributions. As noted just below eqn. (2.13), we only consider the cases in which the photon and the hadron helicities are separately conserved. This is likely to be a good approximation for the proton, but for the photon, there is presumably further contribution from the $(\pm, \mp)$ helicity combinations. These do not mix with the $(\pm, \mp)$ helicity combinations considered here, and so the resulting amplitude would add in quadrature with our amplitude. Hence the dips would be less prominent.

This point would be interesting to test experimentally, though unlikely in the existing experimental environment. A more plausible quantity to measure would be the helicity of the baryons in the pair production of unstable baryons. This would be interesting and the measurement should shed light onto some of the points discussed above, for which we do not yet have a conclusive statement.

3.2 $\gamma\gamma \to K^+K^-$

Let us now turn our attention to the $\gamma\gamma \to K^+K^-$ process.

We show the calculated $\gamma\gamma \to K^+K^-$ cross section in figs. 8 and 9.

As noted in sec. 2.4, we estimate the coupling of the f trajectory by means of Regge factorization using the $pK^\pm$ scattering total cross sections, and estimate the f' trajectory by multiplying the resulting $\overline{\beta}$ by $1/2.5$ to account for the charge of the s-quark. This last procedure is highly arbitrary, but we see that the resulting normalization and the peak positions are roughly in agreement with the data.

There is one unwelcome feature here that the fall-off is more rapid in the calculation than real data. One possible reason is the onset of the perturbative contribution, but there may be further contribution from inaccurate parametrization of the resonances and perhaps extra resonances.
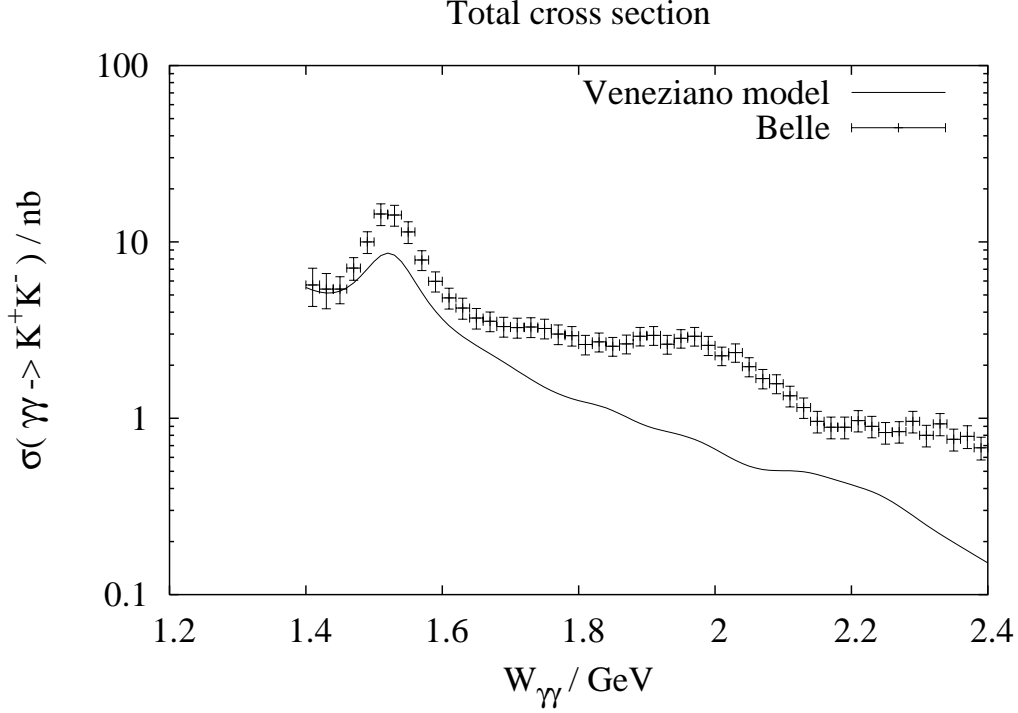
Total cross section

Figure 8: The $\gamma\gamma \to K^+ K^-$ total cross section, in the region $|\cos\theta^*| < 0.6$, calculated using the Veneziano model. The vertical error-bars on the Belle data-points represent the sum in quadrature of the statistical and systematic errors.

The supposition of extra resonances indeed seems quite compelling from the richness of mesonic resonances in this region. Otherwise, the near-flat structure seen in real data between 1.6 GeV and 2 GeV, and between 2.2 GeV and 2.4 GeV, is difficult to explain.

There is further support to this hypothesis from the angular distribution in the sense that we expect perturbative contribution to be predominantly in the forward direction, whereas the difference between the calculation and the data in reality seems more flat.

The angular distribution again reproduces the general trend of the transition from the resonance picture to the t-channel picture. As seen before in the $\gamma\gamma \to p\bar{p}$ case, the t-channel behaviour seems to set in earlier in the calculation. However, this deficiency is less marked in the present case. We note that the baryonic amplitude written in eqn. (2.2) does not reproduce the correct Regge pole behaviour and has an extra factor of $\sqrt{s}$ in the Regge limit of small t and large s, whereas the mesonic amplitude of eqn. (2.1) has the correct Regge pole behaviour in the Regge limit. This may explain the difference.

If so, an amplitude with the correct Regge behaviour may be able to better reproduce the $\gamma\gamma \to p\bar{p}$ amplitude. We have indeed attempted to do this. Our procedure is based on writing several terms of Veneziano amplitudes that are proportional to $\langle p|(\not{p}_{\gamma_1} - \not{p}_{\gamma_2})|\bar{p}\rangle$ and $\langle p|m_p|\bar{p}\rangle$. We have attained some success, but we hesitate to reproduce the results here as there is too much arbitrariness in this procedure and our findings do not show sufficient improvement that compensates for this arbitrariness. Indeed, we shall see in sec. 3.4 that
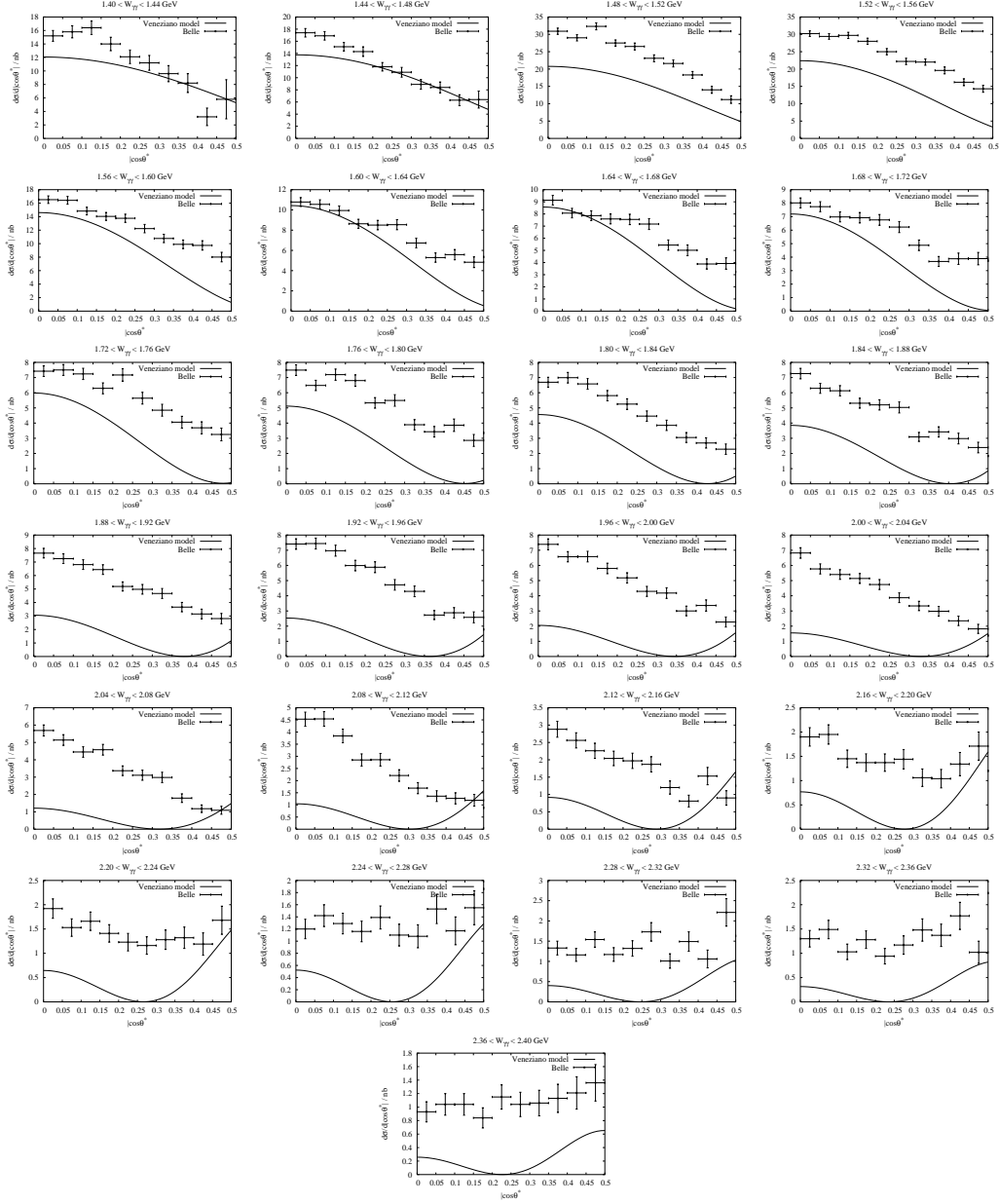
Figure 9: The angular distribution for $\gamma\gamma \to K^+K^-$. The vertical error-bars on the Belle data are statistical only.

the correct Regge expression by itself is insufficient to account for the premature rise of the forward cross section.

We note again that the dips in the $\cos\theta^*$ distribution are in general more pronounced in the calculation than in the real data. However, this is mainly for larger values of $W_{\gamma\gamma}$, where the perturbative contribution is expected to be important.

3.3 $\gamma\gamma \to \Sigma\overline{\Sigma}, \Lambda\overline{\Lambda}$

We make use of the estimations of sec. 2.5 to predict the Σ and the Λ pair production cross
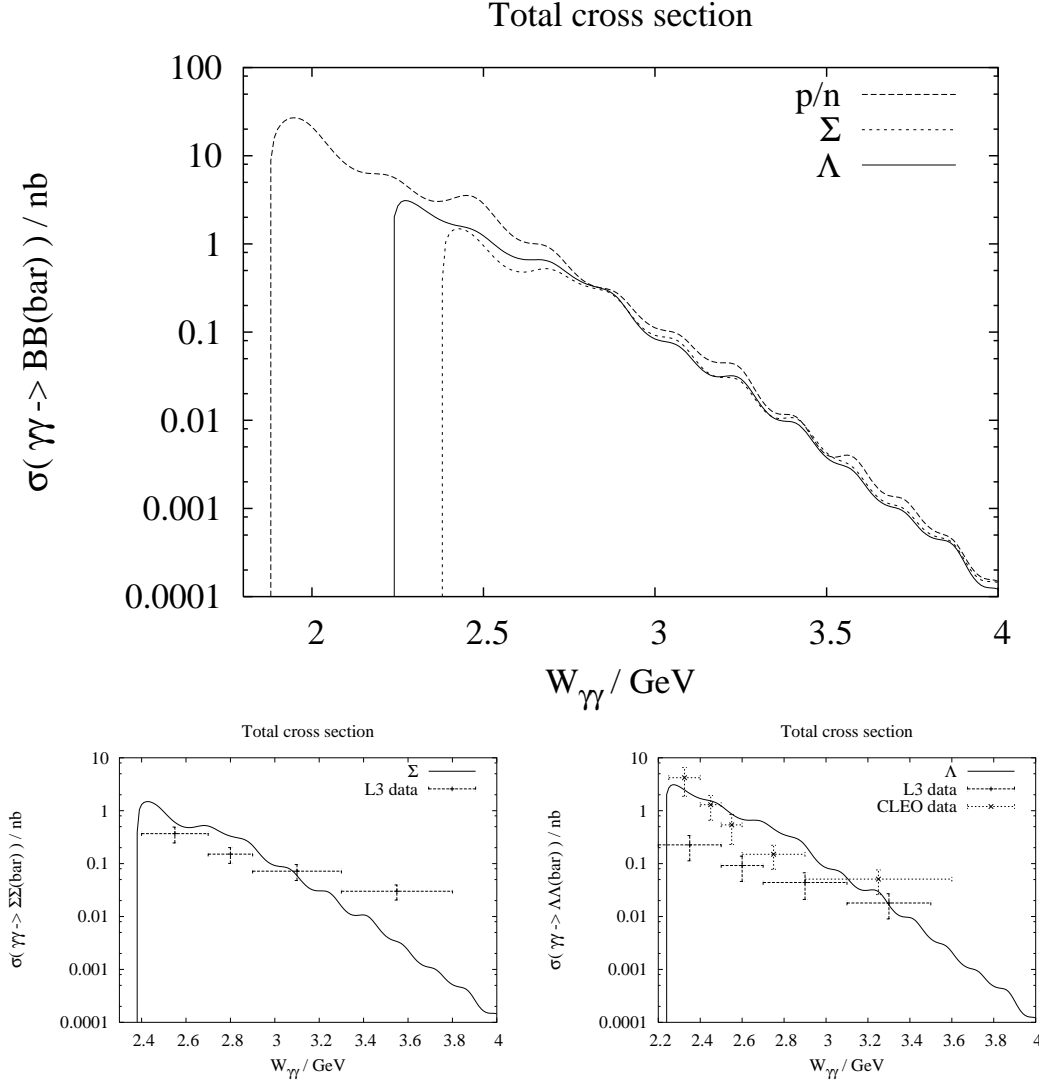
Figure 10: Pair production total cross section, in the region $|\cos\theta^*| < 0.6$, for $p\bar{p}$ (top), $\Sigma\overline{\Sigma}$ (top and bottom left) and $\Lambda\overline{\Lambda}$ (top and bottom right). A comparison is made with the data from L3 [16] and CLEO [17]. The vertical error-bars represent the statistical and systematic errors added in quadrature.

sections in fig. 10. Again, the f' couplings are estimated from the charge of the s-quark. We expect, as noted in sec. 2.5, that the $\Sigma^\pm$ production cross sections are similar to the Σ production cross section.

We should mention that the threshold behaviour is very likely to be incorrect in this case as was in the $p\bar{p}$ case.

A comparison with the measurements at CLEO [17] of $\gamma\gamma \to \Lambda\overline{\Lambda}$ and L3 [16] of $\gamma\gamma \to \Sigma^0\overline{\Sigma^0}, \Lambda\overline{\Lambda}$ shows that the numbers seem to be larger than the L3 cross sections and more in agreement with the CLEO measurement. However, as discussed in sec. 2.5, the normalization for $\Lambda\overline{\Lambda}$ is doubtful because we have not taken into account the isospin-

0 nature of Λ. By comparing the normalizations of the theory predictions for Σ and Λ with the corresponding data from L3, it seems likely that either the normalization for Λ production is too high, or the normalization for Σ production is too low.

Again, above 3 GeV or so, the description is expected to deteriorate because of the effect of perturbative QCD.

3.4 The Regge limit and semi-local duality

One way to understand the above findings is to consider the limiting expressions given in eqns. (2.4) and (2.5).

As stated before, photoproduction reactions are not described well in the Regge pole picture. However, the use of the Veneziano model is motivated by the resonance-pole duality picture, which in turn is implied by the finite-energy sum rules [19].

Semi-local duality is a generalization of the finite-energy sum rules. It states that fluctuations due to resonances cancel out when integrated over a finite energy range and the average is given by the Regge expression, such as eqn. (2.4).

A comparison of eqn. (2.5) with data shows that there is in fact not much difference between the result of this expression and the result of the full Veneziano expression given by eqn. (2.2). However, all the resonances are smoothed out so that the curve is more in agreement with experimental data, and the angular distribution for low $W_{\gamma\gamma}$ is more flat contrary to the real data.

The central peak is still visible for low $W_{\gamma\gamma}$. This is interesting, since Regge expression is merely a sum of u- and t-channel terms. As we have mentioned before, the central peak is best understood to be due to the s-channel resonance behaviour. This is presumably best understood to be an outcome of duality. The sum of t-channel poles is in fact equivalent to the sum of s-channel resonance terms.

It is tempting to compare the data against the correct Regge expression, namely:

$$A(\gamma\gamma \to B\overline{B}) = \frac{\overline{\beta}}{\pi}\Gamma\left(\frac{1}{2} - \alpha_t(t)\right)(-\alpha_s(s))^{\alpha_t(t)} + (t \leftrightarrow u).$$

(3.1)

The form of the equation is similar to eqn. (2.4), and differs from eqn. (2.5) by a factor which is roughly $\sqrt{-s}$. The normalization is different from before, and $\overline{\beta}$ must be determined by other means, such as by fitting with the data. We find that $\overline{\beta}$ needs to be approximately doubled in order to achieve correct normalization.

We have calculated the result of this expression and find again that there is not much difference between this and the other two expressions. The total cross section is shown in fig. 11, and is in good agreement with the data above the threshold region and below about 3 GeV. The angular distribution is in better agreement with the data than that calculated using the wrong Regge limit of eqn. (2.5). The secondary dips that are present above 2.8 GeV in fig. 7 are not seen in this case.

What these observations seem to imply is that the wrong Veneziano expression of eqn. (2.2) nevertheless works, because the amplitude looks like the correct baryonic amplitude whose limiting behaviour is given by the Regge expression of eqn. (3.1).
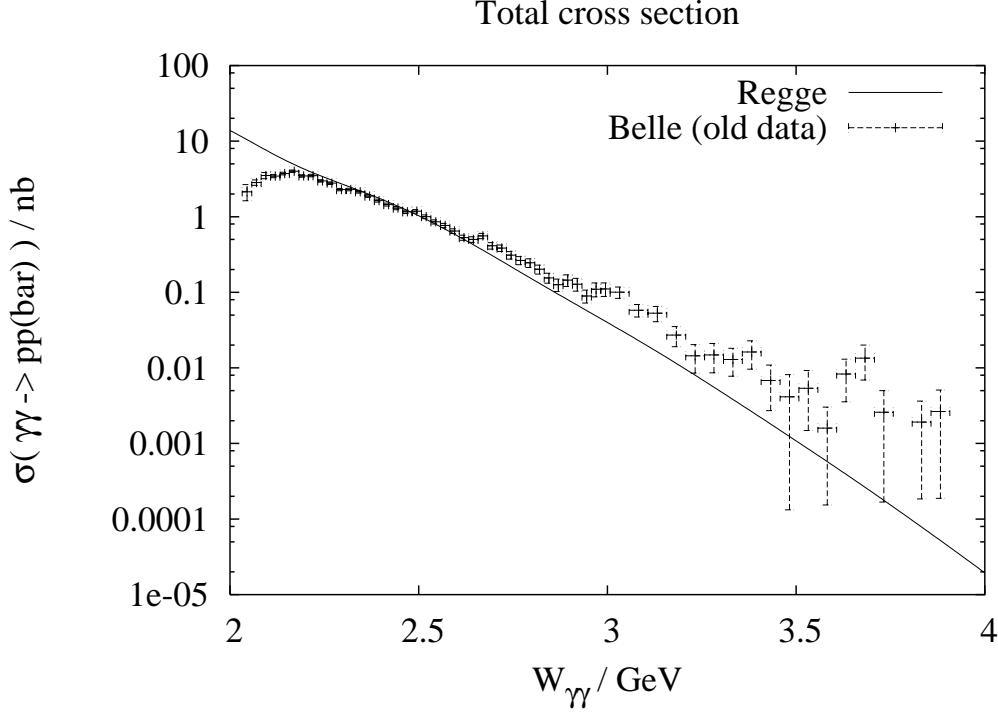
Figure 11: The $\gamma\gamma \to p\bar{p}$ total cross section in the region $|\cos\theta^*| < 0.6$, calculated using the Regge limiting expression. The Belle result is shown for comparison. The vertical error-bars represent the statistical uncertainty only.

Even so, the question remains as to why the Regge approach works in this case in the central region, when it is known that photoproduction and baryon-exchange reactions are not described well by the Regge pole picture.

4. Conclusions

We have shown that an approach based on the resonance-and-pole picture, making use of the Veneziano model, provides a sound description of the exclusive hadron-pair photoproduction processes both for mesons and baryons.

We have concentrated on the case of $\gamma\gamma \to p\bar{p}$ and $\gamma\gamma \to K^+K^-$ at Belle.

For $\gamma\gamma \to p\bar{p}$, there is no adjustable parameter after normalizing the coupling constant against the γp total cross section. For $\gamma\gamma \to K^+K^-$, we obtained the coupling constant using Regge factorization and estimated the contribution of the f' resonances by the charge-counting argument. We also provided rough estimates based on the additive quark rule for $\gamma\gamma \to \Sigma\overline{\Sigma}$ and $\gamma\gamma \to \Lambda\overline{\Lambda}$. For the last of these, we noted that there is a problem due to the isospin-0 nature of Λ.

Based on the suppression of the coupling of the a trajectory to the proton, which follows from the similarity of the $pp, \bar{p}p$ and the $pn, \bar{p}n$ cross sections, we predict that the $\gamma\gamma \to n\bar{n}$ cross section is similar to the $p\bar{p}$ cross section, and that there would be little difference between Σ^-, Σ^0 and Σ^+.

A notable attraction of this procedure is that the transition from the resonance picture with a central peak to the t-channel forward dominance follows naturally. On the other hand, we know that the simple Regge pole picture, inherent in the Veneziano model, fails for photoproduction, and possibly as a result, the t-channel behaviour seems to set in earlier than in the real data. This is particularly so for the case of $\gamma\gamma \to p\bar{p}$ for which the Veneziano amplitude does not have the correct Regge pole behaviour.

We have made a comparison against the correct limiting expression as expected in the Regge picture, and found that the resulting distributions are similar to that obtained using the full Veneziano amplitude except that the resonance peaks are smoothed out. This similarity of the Veneziano amplitude and the correct Regge limiting expression provides partial answer as to why our method, which is based on a formula that does not have the correct Regge limit, nevertheless works. However, it is not clear why the expression for the Regge limit works, when it is known that in photoproduction, the Regge pole picture fails in the Regge limit.

The dips in the angular distribution are more pronounced compared with the real data. We have suggested a possible reason for this, based on the contribution of different helicity components.

When the transverse momentum is large, perturbative QCD is expected to become the more dominant mode of interaction. We do not have a prescription to incorporate this contribution.

We finally note that final states with more than two hadrons can also be estimated in this framework simply by adopting the multi-meson generalizations [20] of the Veneziano model. This should be applicable again in the central region before the onset of perturbative QCD. This would induce one free parameter, namely the normalization, which we presently do not know how to estimate.

Acknowledgments

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